Inequality involving triangles
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Let $x,y,z > 0$ be positive real numbers and S be the area of the triangle ABC with side lengths a,b,c.
Prove that $((x+y)/z)ab + ((y+z)/x)bc + ((z+x)/y)ca \geq 8\sqrt{3}\,S$
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$$\sum \frac{x+y}{z}ab \geq 8\sqrt{3}\,S.$$

Assuming $x+y+z = 1$ we obtain $\sum \frac{x+y}{z}ab = \sum \frac{1-z}{z}ab = \sum \frac{ab}{z} - (ab+bc+ca)$.
Since by Cauchy Inequality $\sum \frac{ab}{z} = (x+y+z)\sum \frac{ab}{z} \geq \left(\sum \sqrt{ab}\right)^2$ remains to prove inequality $\left(\sqrt{ab}+\sqrt{bc}+\sqrt{ca}\right)^2 - (ab+bc+ca) \geq 8\sqrt{3}\,S \Leftrightarrow \sqrt{abc}\left(\sqrt{a}+\sqrt{b}+\sqrt{c}\right) \geq 4\sqrt{3}$
(**1**) $\sqrt{abc}\left(\sqrt{a}+\sqrt{b}+\sqrt{c}\right) \geq 3 \cdot 16S^2 = 3\Delta(a^2,b^2,c^2)$,
where $\Delta(x,y,z) := 2xy+2yz+2zx-x^2-y^2-z^2$. [**1**].
For further we need the following auxiliary inequality:
(**2**) $\Delta^2\left(a,b,c\right) \geq 3\Delta(a^2,b^2,c^2)$.
Let $x := p-a, y := p-b, z := p-c$, where p is semiperimeter of $\triangle ABC$, and let $p := xy+yz+zx, q := xyz$. Assuming $s = 1$we obtain:
$x,y,z > 0, x+y+z = 1, a = 1-x, b = 1-y, c = 1-z,\ S = \sqrt{q},\ ab+bc+ca = 1+p$,
$\Delta\left(a,b,c\right) = 4(ab+bc+ca)-(a+b+c)^2 = 4p,\ \Delta(a^2,b^2,c^2) = 16S^2 = 16q$,
and inequality (**2**) becomes $16p^2 \geq 3 \cdot 16q \Leftrightarrow p^2 \geq 3q$ where latter inequality is inequality $(xy+yz+zx)^2 \geq 3xyz(x+y+z)$ in p,q-notations with normalization by $x+y+z = 1$.
Thus, to complete the solution suffice to prove inequality
(**3**) $\sqrt{abc}\left(\sqrt{a}+\sqrt{b}+\sqrt{c}\right) \geq \Delta^2\left(a,b,c\right)$.
Let R,r,F and s be, respectively, circumradius, inradius,area and semiperimeter of the triangle with sidelengths $\sqrt{a}, \sqrt{b}, \sqrt{c}$ (easy to check that these three numbers satisfies to triangle inequalities). Then $\sqrt{abc} = 4R \cdot F, \sqrt{a}+\sqrt{b}+\sqrt{c} = 2s$,
$\Delta\left(a,b,c\right) = 16F = 16rs$ and inequality (**3**) becomes $4R \cdot F \cdot 2s \geq 16F^2 \Leftrightarrow$
$Rs \geq 2F = 2rs \Leftrightarrow R \geq 2r$ (Euler's Inequality).

[**1**] Arkady Alt, Geometric Inequalities with polynomial 2(xy+yz+zx)-(x^2+y^2+z^2.), OCTOGON Mathematical Magazine Vol.22,n.2,2014, pp.728-741
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